Mentioning and going beyond measures

Aristo Tacoma, May 3, 2019

In looking to the original papers by L.E.J.Brouwer, one of his proposals I agree to, namely what he said about 'clear ideas'. (Some other proposals of his I don't not agree to.) Put simply, he proposed that mathematics can only have meaning and lead to good results if in every step its results are based on clear ideas. That's a lot of a demand, and he was well aware that with such a demand, there is a lot of groundwork to do.

One of the most enduring set of complications and confusing and, indeed, unclear ideas, in the foundation of mathematics concern the notion of infinity, and its many related notions. This I have talked about in various other texts.

I wish to introduce a possible way to talk about infinity which may be of interest to those who admit that the concept is very complicated indeed. This way will connect to the proposal of the necessity to have clear ideas all along. At the moment of writing this, it feels like a fumbling, but the intuition is that it is the right type of fumbling. Here we go:

Let us imagine a context of some sort of logic or mathematics or programming in which we will use the verb "mention" in particular when we have both a clear idea and also put words (or something like words, it can be formal signs) to it. The English word 'mention' clearly shares a root with the word 'mental' and so we can associate this, again, with the notion of 'clear ideas', and to have a mind about something--but also to put this idea into words.

In passing, let us note that an algorithm, even if implemented and performed on a physical computer, is, in a sense, only an algorithm if we mention it to be such (a point elaborated in earlier decades by thinkers such as John R. Searle). Otherwise, we have just a machine that does something by means of such as electronical interactions. To say that such and such 'is an algorithm' is to describe the behaviour of something as taking place in accordance with certain rules. We may nod and agree to this. But unless the description is made at a mental level, there is just the behaviour and that behaviour admits of other types of description.

More generally, we can mention that something has a measure. A measure involves a comparison at a mental level. And let us remind ourselves here that a dictionary can tell that the word 'immeasurable' is one of the synonyms with 'infinity'.

When we look at some of the most well-known numbers, such as 1, 2, and 3, and the well-known idea of 'adding 1', so that we can say, add 1 to 1 and we get 2, and add 1 to 2 and we get 3, then we are, clearly, having measures,-- in the sense that we have a clear sense of grasp of what is going on.

A vaguely related word is that of 'memory'. To not have a memory is related to the idea of 'forget'. In asking, 'Did you forget it?' the answer 'Forgot what?' may be, then, more convincing in the positive sense than the answer 'yes'. The answer 'yes' may indicate that the person is still remembering it but trying to be at a distance from it. The answer 'no' can, however, be entirely clear. The best answers to the question, 'Did you forget it?', then, go along these two lines: "Forgot what?", or, "No.".

Let us consider then, that in looking at the conventional 'start' of the set of socalled 'natural numbers', beginning with 1, then adding 1 to 1 so we get 2, and adding 1 to 2 so we get 3, we are at liberty to say: "We have a measure". We are looking at the algorithm. We are mentioning the measure.

As I have pointed out repeatedly, something very radical happens when we apply the notion of 'et cetera'. Let us therefore try to put something of this into words we have just used.

Please listen to these two sentences and be aware of the ideas associated with them:
"We have a measure."
"We have let go of the measure."

In looking at 1, 2 and 3, we have a measure. Let go of the measure. In asking the question of ourselves, 'Did we let go of the measure?' we can, better than say, "yes", ask with a question back: "Let go of which measure?"

In putting some flesh of example to these abstract ideas, let us rewrite a classical anecdote from Buddhism. Two monks, having just made their oaths never to be attached to women, encounter, at a riverside, a young, scantily clad beautiful woman who is clearly afraid of crossing this dangerous river. Both monks know how to cross that river. One of them picks her up and carries her over and puts her gently down on the other side. Half a day later, the other monk asks, "Did you get attached to her?" The first monk answers, "Attached to who?"

Did we mention that we have let go of the algorithm? The right answer, in a way, in case we have let go of the algorithm, is to ask back, "Which algorithm?" And we can say the same about measure. Really to let go of measure is to "forget" the measure.

In this strong sense of letting go, I propose now a certain word-usage:

By "infinite" we mean "to mention letting go of measure".

It comes natural then to suggest this:

By "finite" we mean "to mention measure."

Let us note that the way we use these two words here, it is not just any negation of 'infinite' that leads to 'finite'. It is a particular negation of it, instead of mentioning letting go of measure, we are mentioning measure. The negation is whether or not "letting go" is included in the idea.

In looking at the idea of 1, then adding 1 to 1 to produce 2, and adding 1 to 2 to produce 3, we are having something finite. We are mentioning measure. To use that other important concept, we have s something like an algorithm, or something algorithmic.

In wanting to go beyond and look at a more general understand of every number, we have to ask ourselves: did you let go of the measure? And this we have only faithfully done in our mind when we could ask back, 'which measure?'. To mention that we have let go of measure in this way is what we mean by 'infinite'. We could possibly coin a word in contrast to 'algorithmic' like 'ingorithmic' to indicate the same type of word-pair as 'finite' and 'infinite'.

Imagine that we begin, not this time with 1, but with 2, and that we added 2, rather than 1, so that the next number is 4, and we add 2 to this so that the next number is 6. We have something like a 'measure' or an 'algorithm' here, but it is a different one than above (in some regards).
So we mention that we have a measure: that is finite.

Let go of that measure. When we ask ourselves, then, 'did we let go of that measure?' then we are having the clearest idea if we are inclined to ask back, 'which measure?'. And in this sense we are mentioning that we have let go of measure: the infinite. But by having a vague sense of just what measure we just have 'forgotten', and comparing this measure, where we add 2 each time, to the measure above where we added 1 each time, we are at liberty to assert: yes, infinite; but it seems to be a different infinite, somehow. (Let us not try to analyze this in conventional unclear terms of 'countability' or such, remember that we are working now towards a vocabulary that can facilitate more clear ideas than that which conventionally has been the case.)

At this point, let us not try to pretend that everything is either entirely a clear idea or the opposite, a perfectly unclear idea. It is easy to talk about a clear idea with 1, 2 and 3, and adding 1 and adding 2. It is more vague to speculate that the infinity we are led to think about in the case of letting go of the measure of adding 1 is a different infinite than we come to in the case of letting go of the measure of adding 2. But it seems to this writer that, once we admit that we are now in the realm of intuition, we are probing into something worth the while probing into, and where we are clearer in saying that the sense of one type of infinity is different than the other type of infinity, than in denying such a possible difference.

The mere fact that we are now talking at this level suggests, I think, that it may well be a fruitful type of language we have just started to use here: that the finite is 'to mention a measure' and that the infinite is 'to mention that we have let go of a measure'. This way of using language has an inbuilt caution in it: we are not led to think that going from the algorithm dealing with a few small numbers and going up and beyond to some kind of infinite set is in any way a smooth, simple or continous process. On the contrary, we are treating the algorithm rather as spice added to a meal where the idea of the infinite comes to us as if from a different type of process, perhaps like tasting the resulting meal.